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PLANAR AND LAMELLAR ANTIFERROMAGNETISMS IN HUBBARD MODELS — COMPLEMENT

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Abstract. This complement to the paper *Planar and lamellar antiferromagnetisms in Hubbard models* [GKU] contains explicit formulæ for the effective potential due to quantum fluctuations, with on-site interactions and at order 4. Since the equations are really unelegant, this complement will not be published.

Notations are those of [GKU] and are not redefined here.

1. Bosons

To write the effective potential in the 4th order, we decompose

$$\Psi^{(4)} = (\Psi^{(4)}_{\{x,y\}}) + (\Psi^{(4)}_{\{x,y,z\}}) + (\Psi^{(4)}_{\{w,x,y,z\}}). \tag{1.1}$$

Analogously to (2.7) in [GKU], we define

$$\phi^{xy,\sigma'}_{xy,\sigma}(g) = \begin{cases} \Phi_x(\hat g_x) + \Phi_y(\hat g_y) - \Phi_x(g_x) - \Phi_y(g_y) & \text{if } \hat g_{x\sigma}, \hat g_{x\sigma'} \geqslant 0 \text{ and } \hat g_{y\sigma}, \hat g_{y\sigma'} \leqslant N \\ \infty & \text{otherwise} \end{cases} \tag{1.2}$$

where

$$\hat g_{x\sigma''} = g_{x\sigma''} - \delta_{\sigma\sigma''} - \delta_{\sigma'\sigma''}$$
$$\hat g_{y\sigma''} = g_{y\sigma''} + \delta_{\sigma\sigma''} + \delta_{\sigma'\sigma''}$$

(we may have $\sigma = \sigma'$); from Assumption 2 in [GKU], $\phi^{xy,\sigma'}_{xy,\sigma}(g) \geqslant 2\Delta_0$. Then

$$\begin{aligned}
\Psi^{(4)}_{\{x,y\}}(g_{\{x,y\}}) = &- \sum_{\sigma\in\Sigma^*} |t_{xy,\sigma}|^4 \Bigg[\frac{(g_{x\sigma}-1)g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma}+2)}{\phi^2_{xy,\sigma}(g)\phi^{xy,\sigma}_{xy,\sigma}(g)} \\
&- \frac{[g_{x\sigma}(g_{y\sigma}+1)]^2}{\phi^3_{xy,\sigma}(g)} - \frac12 \frac{g_{x\sigma}(g_{x\sigma}+1)g_{y\sigma}(g_{y\sigma}+1)}{\phi_{xy,\sigma}(g)+\phi_{yx,\sigma}(g)} \Big\{ \frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{yx,\sigma}(g)} \Big\}^2 + x \leftrightarrow y \Bigg] \\
&- \sum_{\substack{\sigma,\sigma'\in\Sigma^* \\ \sigma\neq\sigma'}} |t_{xy,\sigma}|^2 |t_{xy,\sigma'}|^2 \Bigg[\frac{g_{x\sigma}(g_{y\sigma}+1)g_{x\sigma'}(g_{y\sigma'}+1)}{\phi_{xy,\sigma}(g)\phi^{xy,\sigma'}_{xy,\sigma}(g)} \Big(\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xy,\sigma'}(g)} \Big) \\
&- \frac12 \frac{g_{x\sigma}(g_{y\sigma}+1)g_{x\sigma'}(g_{y\sigma'}+1)}{\phi_{xy,\sigma}(g)+\phi_{xy,\sigma'}(g)} \Big\{ \frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xy,\sigma'}(g)} \Big\}^2 \\
&- \frac12 \frac{g_{x\sigma}(g_{y\sigma}+1)g_{y\sigma'}(g_{x\sigma'}+1)}{\phi_{xy,\sigma}(g)+\phi_{yx,\sigma'}(g)} \Big\{ \frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{yx,\sigma'}(g)} \Big\}^2 + x \leftrightarrow y \Bigg].
\end{aligned} \tag{1.3}$$

To write down the efective potential acting on sets of 3 sites $\{x,y,z\}$, let us decompose further

$$\Psi^{(4)}_{\{x,y,z\}} = \sum_{\sigma\in\Sigma^*} \Psi^{(4),\sigma}_{\{x,y,z\}} + \sum_{\substack{\sigma,\sigma'\in\Sigma^*\\ \sigma\neq\sigma'}} \Psi^{(4),\sigma\sigma'}_{\{x,y,z\}}. \tag{1.4}$$

$$\begin{aligned}
\Psi^{(4),\sigma}_{\{x,y,z\}} = &-|t_{xy,\sigma}|^2|t_{yz,\sigma}|^2\Bigg[\frac{g_{x\sigma}(g_{y\sigma}+1)^2(g_{z\sigma}+1)}{\phi^2_{xy,\sigma}(g)\phi_{xz,\sigma}(g)} + \frac{g_{x\sigma}g_{y\sigma}(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi_{xy,\sigma}(g)\phi_{xz,\sigma}(g)\phi_{yz,\sigma}(g)}\\
&+ \frac{g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma}+2)g_{z\sigma}}{\phi_{xy,\sigma}(g)\phi^{zy,\sigma}_{xy,\sigma}(g)\phi_{zy,\sigma}(g)} + \frac{g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma}+2)g_{z\sigma}}{\phi^2_{xy,\sigma}(g)\phi^{zy,\sigma}_{xy,\sigma}(g)}\\
&- \frac{1}{2}\frac{g_{x\sigma}g_{y\sigma}(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi_{xy,\sigma}(g)+\phi_{yz,\sigma}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{yz,\sigma}(g)}\right\}^2\\
&- \frac{1}{2}\frac{g_{x\sigma}(g_{y\sigma}+1)^2 g_{z\sigma}}{\phi_{xy,\sigma}(g)+\phi_{zy,\sigma}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{zy,\sigma}(g)}\right\}^2\Bigg]\\
&- |t_{xy,\sigma}|^2|t_{xz,\sigma}|^2\Bigg[\frac{(g_{x\sigma}-1)g_{x\sigma}(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi_{xy,\sigma}(g)\phi^{xz,\sigma}_{xy,\sigma}(g)\phi_{xz,\sigma}(g)} + \frac{(g_{x\sigma}-1)g_{x\sigma}(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi^2_{xy,\sigma}(g)\phi^{xz,\sigma}_{xy,\sigma}(g)}\\
&+ \frac{g_{x\sigma}(g_{x\sigma}+1)(g_{y\sigma}+1)g_{z\sigma}}{\phi_{xy,\sigma}(g)\phi_{zy,\sigma}(g)\phi_{zx,\sigma}(g)} + \frac{g^2_{x\sigma}(g_{y\sigma}+1)g_{z\sigma}}{\phi^2_{xy,\sigma}(g)\phi^{zy,\sigma}_{zy,\sigma}(g)}\\
&- \frac{1}{2}\frac{g^2_{x\sigma}(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi_{xy,\sigma}(g)+\phi_{xz,\sigma}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xz,\sigma}(g)}\right\}^2\\
&- \frac{1}{2}\frac{g_{x\sigma}(g_{x\sigma}+1)(g_{y\sigma}+1)g_{z\sigma}}{\phi_{xy,\sigma}(g)+\phi_{zx,\sigma}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{zx,\sigma}(g)}\right\}^2\Bigg]\\
&+ \text{permutations of } (x,y,z).
\end{aligned} \tag{1.5}$$

Here, we defined

$$\phi^{xz,\sigma'}_{xy,\sigma}(g) = \begin{cases} \Phi_x(\hat g_x)+\Phi_y(\hat g_y)+\Phi_z(\hat g_z)-\Phi_x(g_x)-\Phi_y(g_y)-\Phi_z(g_z) & \text{if } \hat g_{x\sigma}, \hat g_{x\sigma'} \geqslant 0 \text{ and } \hat g_{y\sigma}, \hat g_{z\sigma'} \leqslant N\\ \infty & \text{otherwise}\end{cases} \tag{1.6}$$

where

$$\begin{aligned}
\hat g_{x\sigma''} &= g_{x\sigma''} - \delta_{\sigma\sigma''} - \delta_{\sigma'\sigma''}\\
\hat g_{y\sigma''} &= g_{y\sigma''} + \delta_{\sigma\sigma''}\\
\hat g_{z\sigma''} &= g_{z\sigma''} + \delta_{\sigma'\sigma''},
\end{aligned}$$

and

$$\phi^{zy,\sigma'}_{xy,\sigma}(g) = \begin{cases} \Phi_x(\hat g_x)+\Phi_y(\hat g_y)+\Phi_z(\hat g_z)-\Phi_x(g_x)-\Phi_y(g_y)-\Phi_z(g_z) & \text{if } \hat g_{x\sigma}, g_{z\sigma'} \geqslant 0 \text{ and } \hat g_{y\sigma}, \hat g_{y\sigma'} \leqslant N\\ \infty & \text{otherwise}\end{cases} \tag{1.7}$$

where

$$\begin{aligned}
\hat g_{x\sigma''} &= g_{x\sigma''} - \delta_{\sigma\sigma''}\\
\hat g_{y\sigma''} &= g_{y\sigma''} + \delta_{\sigma\sigma''} + \delta_{\sigma'\sigma''}\\
\hat g_{z\sigma''} &= g_{z\sigma''} - \delta_{\sigma'\sigma''}.
\end{aligned}$$

(Both $\phi^{xz,\sigma'}_{xy,\sigma}(g)$ and $\phi^{zy,\sigma'}_{xy,\sigma}(g)$ are greater than $3\Delta_0$, as a result of Assumption 2 in [GKU].)

The term involving different spins is equally beautiful:

$$\begin{aligned}\Psi^{(4),\sigma\sigma'}_{\{x,y,z\}} = -|t_{xy,\sigma}|^2|t_{yz,\sigma'}|^2\Bigg[&\frac{g_{x\sigma}(g_{y\sigma}+1)g_{y\sigma'}(g_{z\sigma'}+1)}{\phi_{xy,\sigma}(g)\phi^{xy,\sigma}_{yz,\sigma'}(g)\phi_{yz,\sigma'}(g)} + \frac{g_{x\sigma}(g_{y\sigma}+1)g_{y\sigma'}(g_{z\sigma'}+1)}{\phi^2_{xy,\sigma}(g)\phi^{xy,\sigma}_{yz,\sigma'}(g)}\\
&+\frac{g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma'}+1)g_{z\sigma'}}{\phi_{xy,\sigma}(g)\phi^{zy,\sigma'}_{xy,\sigma}(g)\phi_{zy,\sigma'}(g)} + \frac{g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma'}+1)g_{z\sigma'}}{\phi^2_{xy,\sigma}(g)\phi^{zy,\sigma'}_{xy,\sigma}(g)}\\
&-\frac{1}{2}\frac{g_{x\sigma}(g_{y\sigma}+1)g_{y\sigma'}(g_{z\sigma'}+1)}{\phi_{xy,\sigma}(g)+\phi_{yz,\sigma'}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)}+\frac{1}{\phi_{yz,\sigma'}(g)}\right\}^2\\
&-\frac{1}{2}\frac{g_{x\sigma}(g_{y\sigma}+1)(g_{y\sigma'}+1)g_{z\sigma'}}{\phi_{xy,\sigma}(g)+\phi_{zy,\sigma'}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)}+\frac{1}{\phi_{zy,\sigma'}(g)}\right\}^2\Bigg]\\
-|t_{xy,\sigma}|^2|t_{xz,\sigma'}|^2\Bigg[&\frac{g_{x\sigma}(g_{y\sigma}+1)g_{x\sigma'}(g_{z\sigma'}+1)}{\phi_{xy,\sigma}(g)\phi^{xz,\sigma'}_{xy,\sigma}(g)\phi_{xz,\sigma'}(g)} + \frac{g_{x\sigma}(g_{y\sigma}+1)g_{x\sigma'}(g_{z\sigma'}+1)}{\phi^2_{xy,\sigma}(g)\phi^{xz,\sigma'}_{xy,\sigma}(g)}\\
&+\frac{g_{x\sigma}(g_{y\sigma}+1)(g_{x\sigma'}+1)g_{z\sigma'}}{\phi_{xy,\sigma}(g)\phi^{zx,\sigma'}_{xy,\sigma}(g)\phi_{xz,\sigma'}(g)} + \frac{g_{x\sigma}(g_{y\sigma}+1)(g_{x\sigma'}+1)g_{z\sigma'}}{\phi^2_{xy,\sigma}(g)\phi^{zx,\sigma'}_{xy,\sigma}(g)}\\
&-\frac{1}{2}\frac{g_{x\sigma}(g_{y\sigma}+1)g_{x\sigma'}(g_{z\sigma'}+1)}{\phi_{xy,\sigma}(g)+\phi_{xz,\sigma'}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)}+\frac{1}{\phi_{xz,\sigma'}(g)}\right\}^2\\
&-\frac{1}{2}\frac{g_{x\sigma}(g_{y\sigma}+1)(g_{x\sigma'}+1)g_{z\sigma'}}{\phi_{xy,\sigma}(g)+\phi_{zx,\sigma'}(g)}\left\{\frac{1}{\phi_{xy,\sigma}(g)}+\frac{1}{\phi_{zx,\sigma'}(g)}\right\}^2\Bigg]\\
&+\text{permutations of } (x,y,z). \quad (1.8)\end{aligned}$$

Here,

$$\phi^{zx,\sigma'}_{xy,\sigma}(g) = \begin{cases}\Phi_x(\hat g_x)+\Phi_y(\hat g_y)+\Phi_z(\hat g_z)-\Phi_x(g_x)-\Phi_y(g_y)-\Phi_z(g_z) & \text{if } g_{x\sigma}, g_{z\sigma'}>0 \text{ and } g_{x\sigma'}, g_{y\sigma}<N\\ \infty & \text{otherwise}\end{cases} \quad (1.9)$$

where

$$\begin{aligned}\hat g_{x\sigma''} &= g_{x\sigma''} - \delta_{\sigma\sigma''} + \delta_{\sigma'\sigma''}\\ \hat g_{y\sigma''} &= g_{y\sigma''} + \delta_{\sigma\sigma''}\\ \hat g_{z\sigma''} &= g_{z\sigma''} - \delta_{\sigma'\sigma''}\end{aligned}$$

($\phi^{zx,\sigma'}_{xy,\sigma}(g) \geqslant 3\Delta_0$).

The effective potential on sets of 4 sites is less cumbersome.

$$\begin{aligned}\Psi^{(4)}_{\{w,x,y,z\}} = -\sum_{\sigma\in\Sigma^*} t_{wx,\sigma}t_{xy,\sigma}t_{yz,\sigma}t_{zw,\sigma}\Bigg[&\frac{g_{w\sigma}(g_{x\sigma}+1)(g_{y\sigma}+1)(g_{z\sigma}+1)}{\phi_{wx,\sigma}(g)\phi_{wy,\sigma}(g)\phi_{wz,\sigma}(g)}\\
&+\frac{g_{w\sigma}(g_{x\sigma}+1)(g_{y\sigma}+1)g_{z\sigma}}{\phi_{wx,\sigma}(g)\phi_{wy,\sigma}(g)\phi_{zy,\sigma}(g)}+\frac{g_{w\sigma}(g_{x\sigma}+1)g_{y\sigma}(g_{z\sigma}+1)}{\phi_{wx,\sigma}(g)[\phi_{wx,\sigma}(g)+\phi_{yz,\sigma}(g)]\phi_{wz,\sigma}(g)}\\
+\frac{g_{w\sigma}(g_{x\sigma}+1)g_{y\sigma}(g_{z\sigma}+1)}{\phi_{wx,\sigma}(g)[\phi_{wx,\sigma}(g)+\phi_{yz,\sigma}(g)]\phi_{yx,\sigma}(g)}&+\frac{g_{w\sigma}(g_{x\sigma}+1)(g_{y\sigma}+1)g_{z\sigma}}{\phi_{wx,\sigma}(g)\phi_{zx,\sigma}(g)\phi_{zy,\sigma}(g)}+\frac{g_{w\sigma}(g_{x\sigma}+1)g_{y\sigma}g_{z\sigma}}{\phi_{wx,\sigma}(g)\phi_{zx,\sigma}(g)\phi_{yx,\sigma}(g)}\Bigg]\\
&+\text{permutations of } (w,x,y,z). \quad (1.10)\end{aligned}$$

2. Fermions

As in the bosonic case, we decompose

$$\Psi^{(4)} = (\Psi^{(4)}_{\{x,y\}}) + (\Psi^{(4)}_{\{x,y,z\}}) + (\Psi^{(4)}_{\{w,x,y,z\}}). \tag{2.1}$$

We define $\phi^{xy,\sigma'}_{xy,\sigma}(g)$, $\phi^{xz,\sigma'}_{xy,\sigma}(g)$, $\phi^{zy,\sigma'}_{xy,\sigma}(g)$, $\phi^{zx,\sigma'}_{xy,\sigma}(g)$ as before, by (1.2), (1.6), (1.7), (1.9) respectively (recall that in the case of fermions, we have $N = 1$).

$$\begin{aligned}\Psi^{(4)}_{\{x,y\}}(g_{\{x,y\}}) = &- \sum_{\sigma\in\Sigma^*} |t_{xy,\sigma}|^4 \Big[-\frac{1}{\phi^3_{xy,\sigma}(g)} + x \leftrightarrow y\Big] \\ &- \sum_{\substack{\sigma,\sigma'\in\Sigma^* \\ \sigma\neq\sigma'}} |t_{xy,\sigma}|^2 |t_{xy,\sigma'}|^2 \Big[\frac{1}{\phi_{xy,\sigma}(g)\phi^{xy,\sigma'}_{xy,\sigma}(g)}\Big(\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xy,\sigma'}(g)}\Big) \\ &\qquad - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g) + \phi_{xy,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xy,\sigma'}(g)}\Big\}^2 \\ &\qquad - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g) + \phi_{yx,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{yx,\sigma'}(g)}\Big\}^2 + x \leftrightarrow y\Big].\end{aligned} \tag{2.2}$$

Again, we decompose

$$\Psi^{(4)}_{\{x,y,z\}} = \sum_{\sigma\in\Sigma^*} \Psi^{(4),\sigma}_{\{x,y,z\}} + \sum_{\substack{\sigma,\sigma'\in\Sigma^* \\ \sigma\neq\sigma'}} \Psi^{(4),\sigma\sigma'}_{\{x,y,z\}}. \tag{2.3}$$

$$\begin{aligned}\Psi^{(4),\sigma}_{\{x,y,z\}} = &-|t_{xy,\sigma}|^2 |t_{yz,\sigma}|^2 \Big[\frac{1}{\phi^2_{xy,\sigma}(g)\phi_{xz,\sigma}(g)} \\ &\qquad - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g) + \phi_{zy,\sigma}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{zy,\sigma}(g)}\Big\}^2\Big] \\ &- |t_{xy,\sigma}|^2 |t_{xz,\sigma}|^2 \Big[\frac{1}{\phi^2_{xy,\sigma}(g)\phi^{zy,\sigma}_{zy,\sigma}(g)} - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g) + \phi_{xz,\sigma}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xz,\sigma}(g)}\Big\}^2\Big] \\ &+ \text{permutations of } (x,y,z).\end{aligned} \tag{2.4}$$

For the term involving different spins, we get

$$\begin{aligned}
\Psi^{(4),\sigma\sigma'}_{\{x,y,z\}} = -|t_{xy,\sigma}|^2|t_{yz,\sigma'}|^2 \Bigg[& \frac{1}{\phi_{xy,\sigma}(g)\phi^{xy,\sigma}_{yz,\sigma'}(g)\phi_{yz,\sigma'}(g)} + \frac{1}{\phi^2_{xy,\sigma}(g)\phi^{xy,\sigma}_{yz,\sigma'}(g)} \\
& + \frac{1}{\phi_{xy,\sigma}(g)\phi^{zy,\sigma'}_{xy,\sigma}(g)\phi_{zy,\sigma'}(g)} + \frac{1}{\phi^2_{xy,\sigma}(g)\phi^{zy,\sigma'}_{xy,\sigma}(g)} \\
& - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g)+\phi_{yz,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{yz,\sigma'}(g)}\Big\}^2 \\
& - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g)+\phi_{zy,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{zy,\sigma'}(g)}\Big\}^2 \Bigg] \\
- |t_{xy,\sigma}|^2|t_{xz,\sigma'}|^2 \Bigg[& \frac{1}{\phi_{xy,\sigma}(g)\phi^{xz,\sigma'}_{xy,\sigma}(g)\phi_{xz,\sigma'}(g)} + \frac{1}{\phi^2_{xy,\sigma}(g)\phi^{xz,\sigma'}_{xy,\sigma}(g)} \\
& + \frac{1}{\phi_{xy,\sigma}(g)\phi^{zx,\sigma'}_{xy,\sigma}(g)\phi_{xz,\sigma'}(g)} + \frac{1}{\phi^2_{xy,\sigma}(g)\phi^{zx,\sigma'}_{xy,\sigma}(g)} \\
& - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g)+\phi_{xz,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{xz,\sigma'}(g)}\Big\}^2 \\
& - \frac{1}{2}\frac{1}{\phi_{xy,\sigma}(g)+\phi_{zx,\sigma'}(g)}\Big\{\frac{1}{\phi_{xy,\sigma}(g)} + \frac{1}{\phi_{zx,\sigma'}(g)}\Big\}^2 \Bigg] \\
& + \text{permutations of } (x,y,z). \quad (2.5)
\end{aligned}$$

Finally, for four distinct sites, we have

$$\begin{aligned}
\Psi^{(4)}_{\{w,x,y,z\}} = -\sum_{\sigma\in\Sigma^*} t_{wx,\sigma}t_{xy,\sigma}t_{yz,\sigma}t_{zw,\sigma} \Bigg[& \frac{1}{\phi_{wx,\sigma}(g)\phi_{wy,\sigma}(g)\phi_{wz,\sigma}(g)} \\
& + \frac{1}{\phi_{wx,\sigma}(g)\phi_{wy,\sigma}(g)\phi_{zy,\sigma}(g)} + \frac{1}{\phi_{wx,\sigma}(g)[\phi_{wx,\sigma}(g)+\phi_{yz,\sigma}(g)]\phi_{wz,\sigma}(g)} \\
+ \frac{1}{\phi_{wx,\sigma}(g)[\phi_{wx,\sigma}(g)+\phi_{yz,\sigma}(g)]\phi_{yx,\sigma}(g)} & + \frac{1}{\phi_{wx,\sigma}(g)\phi_{zx,\sigma}(g)\phi_{zy,\sigma}(g)} + \frac{1}{\phi_{wx,\sigma}(g)\phi_{zx,\sigma}(g)\phi_{yx,\sigma}(g)} \Bigg] \\
& + \text{permutations of } (w,x,y,z). \quad (2.6)
\end{aligned}$$

References

[GKU] Ch. Gruber, R. Kotecký and D. Ueltschi, *Planar and lamellar antiferromagnetisms in Hubbard models*, preprint (1999)