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REFLECTION POSITIVITY, GAUSSIAN DOMINATION, AND PHASE TRANSITIONS IN CLASSICAL AND QUANTUM SYSTEMS

Reflection positivity (RP) is a technique that has been used in statistical mechanics since the end of 1970s. There are two principal conclusions that RP allows one to make: infrared bounds and chessboard estimates. In the works of Dyson, Fröhlich, Israel, Lieb and Simon, these two have been used to establish phase transitions in many classical (continuum spin) and quantum systems (e.g., classical Heisenberg ferromagnet, the quantum XY model, etc). Some nice extensions followed (e.g., a proof of order-disorder transition in the Potts model by Kotecky & Shlosman) but, overall, the technique seemed to have run out of useful applications.

This has changed in recent years when several novel applications of RP have been developed. Here is just one example: RP can be used to show that if a model undergoes a first order-phase transition in the mean-field theory, a similar transition occurs in the actual model provided either the dimension is sufficiently large or the interaction is sufficiently spread out. The condition of RP is of course somewhat restrictive (and so its conclusions are not so robust as those of other techniques, such as contour expansions), but it enables elegant proofs and, furthermore, includes systems where no other methods have yet been made to work.

In my lectures I will attempt to give a thorough introduction to reflection positivity, chessboard estimates and infrared bounds. By means of examples, I will explain how these are used to conclude phase transitions. Then I will review some of the more recent developments including both classical and quantum systems. I plan to write a series of notes which should be available for download towards the end of August.