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LOCALIZATION-DELOCALIZATION PHENOMENA IN RANDOM POLYMER MODELS (SUMMARY OF LECTURES)

I will give an introduction to some models of polymers in interaction with spatial defects. The typical situation to keep in mind is that of a directed polymer in $(d+1)$ dimensions (say, a random walk), interacting with a 1-dimensional defect line. The interaction can be either attractive or repulsive. This kind of models arises quite naturally in physics and biophysics; they are used, for instance, to describe wetting phenomena in 2-dimensional Ising systems, or the denaturation transition of DNA.

In the thermodynamic limit where the length N of the polymer tends to infinity, by tuning the interaction parameters these systems undergo a localization-delocalization transition: in the localized phase the polymer stays close to the defect (even for $N \to \infty$), while in the delocalized one it gets far away from it (say, it distance grows like $\sqrt{N}$). In the phase diagram, the two phases are separated by a critical line.

The situation is particularly interesting when quenched randomness is present: in this case, the defect line is inhomogeneus, i.e., it is attractive in some space regions and repulsive in others. The localization-delocalization transition, which in homogeneous (i.e., non-disordered) models is the effect of an intuitive energy-entropy competition, is strongly influenced in the inhomogeneous case by the presence of rare regions with atypical disorder.

The most interesting open - or only partially answered - questions concern the identification of the critical line and of the critical exponents of the transition, in presence of randomness.

The subject is quite actively studied both by theoretical phisicists and by probabilists, and has recently witnessed interesting developments.

Tentatively, I will discuss the following points:

- The models: motivations, applications and examples (pinning and wetting models, the Poland-Scheraga model of denaturation transition, heteropolymers at selective interfaces)
- Existence of the free energy and its independence of disorder realization (with a brief discussion of concentration-of-measure issues)
- The localization-delocalization phase transition and its order parameter

- The “explicit” solution of the homogeneous pinning model
- Path properties (i.e.: how sharply can one say that the polymer is localized in the localized phase, and delocalized in the delocalized phase?)
- Effect of disorder on the phase transition: the heuristic Harris criterion and some rigorous results

An introduction to the subject can be found in the first few chapters of the lecture notes by G. Giacomin, *Localization phenomena in random polymer models (2004)*, available online:
`www.proba.jussieu.fr/pageperso/giacomin/pub/publicat.html`
On the other hand, most recent results discussed in the minicourse will be extracted from the following papers (none of which is a review article):

- G. Giacomin, F.L. Toninelli, *Smoothing of Depinning Transitions for Directed Polymers with Quenched Disorder*, Phys. Rev. Lett **96**, 060702 (2006), `cond-mat/0510472`
- G. Giacomin, F.L. Toninelli, *The localized phase of disordered copolymers with adsorption*, ALEA **1**, 149-180 (2006), `math.PR/0510047`
- G. Giacomin, F.L. Toninelli, *Smoothing effect of quenched disorder on polymer depinning transitions*, Commun. Math. Phys, to appear, `math.PR/0506431`
- G. Giacomin, F.L. Toninelli, *Estimates on path delocalization for copolymers at selective interfaces*, Probab. Theory Rel. Fields **133**, Number 4, 464-482 (2005), `math.PR/0502304`