Vopěnka's Attempt to Reconstruct the Natural World

Štěpán Holub, Faculty of Mahematics and Physics, Charles University, Prague

Horizons, October 25, 2025

Main concern

Vopěnka's two declared sources of inspiration

- set theory
- (Husserl's) phenomenology

are incompatible.

Mixing the two sources

- is philosophically dubious
- obfuscates the contribution to foundational questions

(Re)discovering the foundational context

The AST is not the singularity it is sometimes taken for, but rather a part of well discernible currents in mathematics.

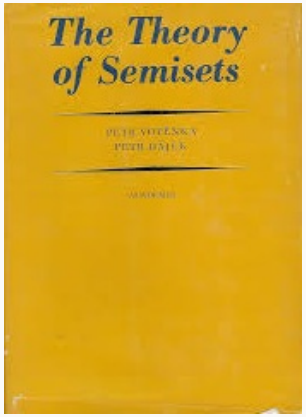
(Z. Hanikova, Vopěnka's Alternative Set Theory in the Mathematical Canon of the 20th Century, 2022)

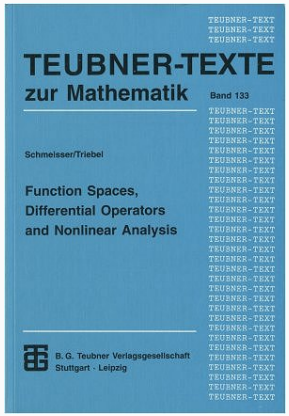
Main concern

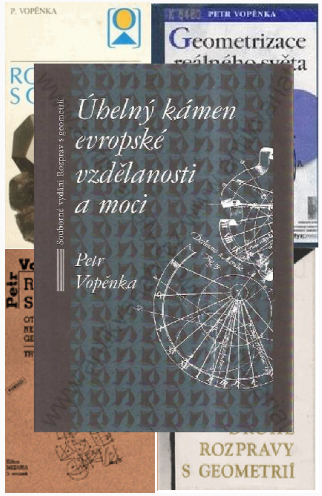
In the book *Mathematics in the Alternative Set Theory*, I overestimated the readers. I believed that the philosophy of Alternative Set Theory would be immediately obvious to them. However, most mathematicians are not able to carry out a phenomenological critique of classical ideas, let alone realize its consequences.

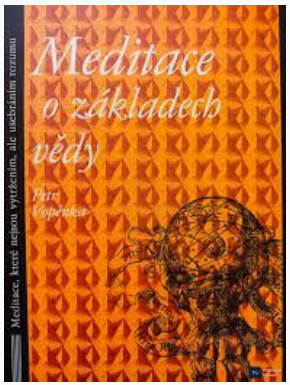
(Proceedings of Symposioum on AST, Stará Lesná, 1989)

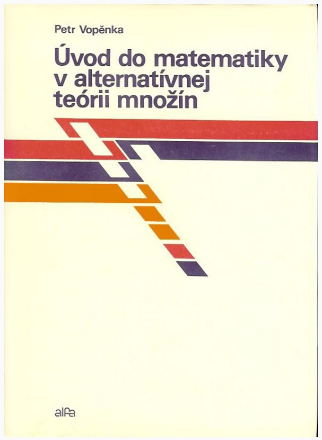
- Why phenomenology?
- What crisis?
- Why set theory?
- Can AST save vagueness?

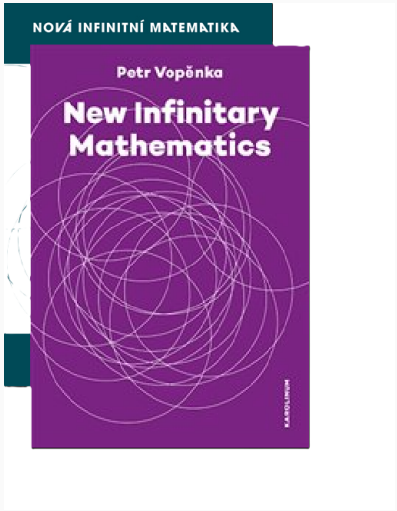

The Theory
of Semisets

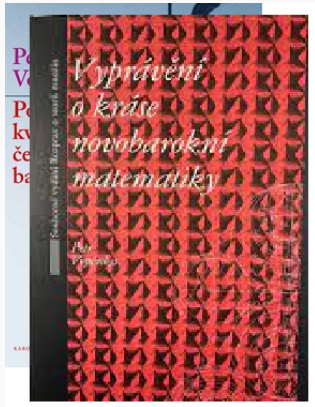

TEUBNER-TEXTE
zur Mathematik
Band 133
Schmeisser/Triebel
Function Spaces,
Differential Operators
and Nonlinear Analysis
B. G. Teubner Verlagsgesellschaft
Stuttgart · Leipzig


Úhelný kámen
evropské
vzdělanosti
a moci
Petr
Vopěnka
ROZPRAVY
S GEOMETRIÍ


Meditace
o základech
vědy
Petr
Vopěnka


Petr Vopěnka
Úvod do matematiky
v alternatívnej
teórii množín
alfa


NOVÁ INFINITNÍ MATEMATIKA
Petr Vopěnka
New Infinitary
Mathematics
KAROLINUM


Vyprávění
o kráse
novobarokní
matematiky

Monographs on Alternative Set Theory

- The Theory of Semisets, with P. Hájek, Prague 1972
- Mathematics in the Alternative Set Theory, Teubner Verlag, Leipzig, 1979. (English)
- Introduction to Mathematics in the Alternative Set Theory, Alfa, Bratislava, 1989 (Slovak, transl. by Pavol Zlatoš)
- New Infinitary Mathematics, Karolinum, 2014 (Czech), 2022 (English, ed. Alena Vencovská)

Petr Vopěnka and transcendental phenomenology

- I was not satisfied with [the current state] of mathematics, and I began working on an alternative set theory as early as in 1973, influenced by Husserl's phenomenology. I was convinced, that it is precisely phenomenology that can provide answers to several questions concerning foundations of mathematics. (Epilog to "A tale about the beauty of neo-baroque mathematics", 2004)
- Our capacity for observation and distinction is limited by the horizon in all directions. Needless to say, this applies not only to optical observation; the horizon is understood in the sense of E. Husserl's "Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie". (Teubner, 1979)

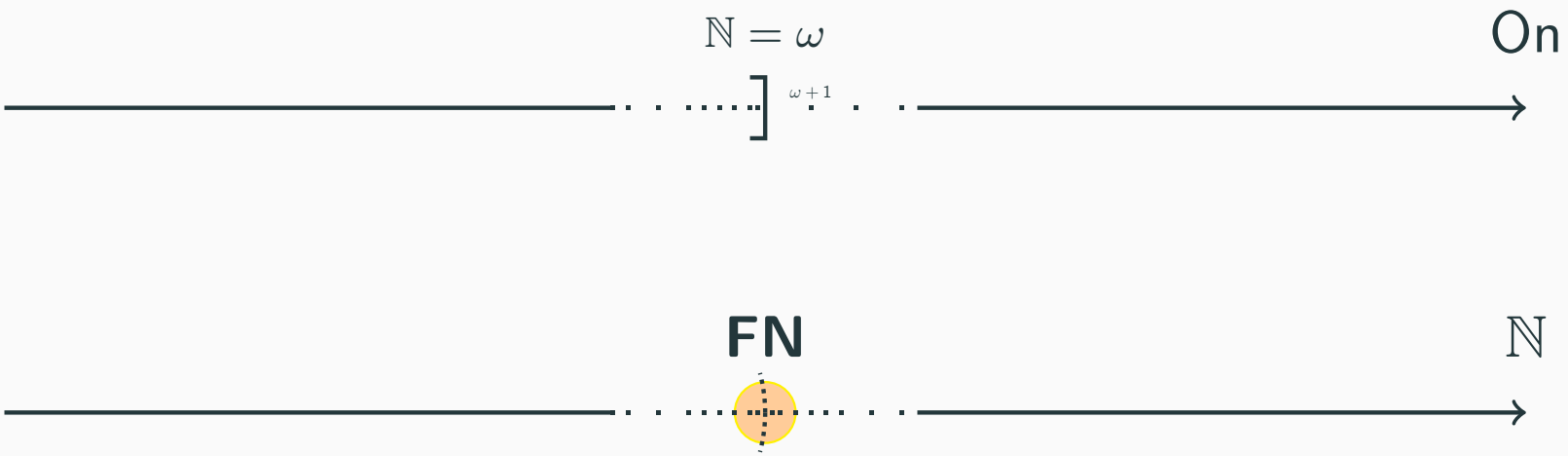
Scholion for phenomenologists

The answer to the question of whether the domain of all natural numbers is actualisable can only be sought in the mysterious understanding we have for the passing of natural numbers towards infinity. For this purpose, we have to cleanse this understanding as far as possible from all additional interpretations which obscure this passing of natural numbers towards infinity.

If we carry out such a purification, or the phenomenological ἐποχή as Edmund Husserl would call it, we have no option but to state that the purified understanding of the passing of natural numbers towards infinity does not give us the slightest cause to shorten the domain of all natural numbers. On the contrary, the raw infinity present in this domain a priori excludes all the obstacles which could hinder natural numbers as they flee towards infinity.

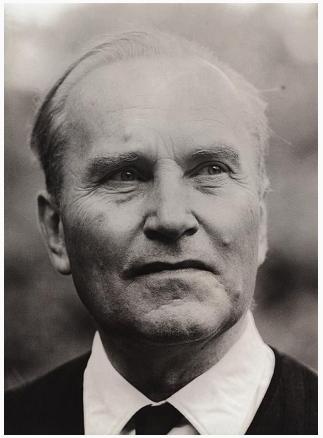
(New Infinitary Mathematics, 2014)

Cantor vs. AST



Why Husserl?

- Husserl born in Moravia, mentored in Berlin by T.G.Masaryk
- *Krisis* closely related to lectures given in Prague in autumn 1935 by invitation from *Cercle philosophique de Prague*
- lectures organized by Jan Patočka secretary of the Czech section of *Cercle* (Archive at CTS)
  - in 1933 student of Husserl and Heidegger (contacts with of Fink, Klein)
  - 1936 Natural world as a philosophical problem
  - 1972 fired from Charles University
  - 1973-1976: flat seminars reading of *Krisis*
  - 1977 one of first speakers of *Charta 77* dies after a police interrogation



- Jiří Polívka: student of Patočka, assistant of Petr Vopěnka "My colleague J. Polívka helped to improve various motivations."
- Vopěnka never mentions Patočka

What Krisis?

Edmund Husserl

Diagnosis: modern science lost contact with its foundation in human life, and thereby lost its ultimate sense.

Remedy: transcendental turn gains apodictic field of pure consciousness which allows to build philosophy as a rigorous science

Petr Vopěnka

Diagnosis: Cantorian theory of actual infinity lost contact with natural world and distorted mathematics

Remedy: Alternative/New set theory

Far reaching claims

Because of its impassability, death is the model according to which classical mathematics developed its concept of the fixed and impassable horizon laying in the absolute infinity, which is represented by classical natural numbers. At the same time [...] the role of the absolute infinity is to prevail over death. (Bratislava, 1989)

Why set theory?

No phenomenological justification for the choice of the set theory.

- Due to the central role of set theory in mathematics it seems reasonable to begin by presenting a new set theory as a possible basis for mathematics, just as contemporary mathematics is based on Cantor set theory. (Teubner, 1979)

Other foundational alternatives?

According to Vopěnka rejecting the law of excluded middle is *incompatible* with the guiding purpose of set theory to actualize infinite domains.

Guiding principles of the set theory (Bratislava, 1989)

- to interpret the world so that as many phenomena as possible are declared to be objects
- to reconstruct the structure of the world in the universe investigated by the set theory (revealed relatively late)
- to explore and understand infinity
- to interpret every infinity as multiplicity (repeated references to Bolzano)

Main additional guiding principles of the AST

- to interpret infinity from the phenomenon of indistinctness (non-sharpness, vagueness)

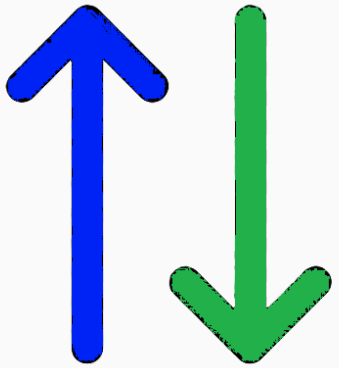
  NB: classical mathematics does the opposite: it interprets the non-sharpness using absolute infinity.

Four kinds of collections

- **class**: (exhausted) collection of (already existing) objects; itself an object
- **domain** covering even not yet existing/created objects; not objects in general (potential infinity)
- **set**: sharply defined class (classically finite)
- **semisets**: non-sharply defined subclass of a set (natural infinity)

[1] Sharpness refers to distinctness, clarity and definiteness, . . . , in brief, the ancient perfection of geometric objects studied in Euclid's *Elements*.

set theory (AST/Cantor)



world (natural/distorted)

A world in which the beautiful, indeterminate shapes and colors appearing in a blossoming meadow are not worthy of the attention of the science of this world.

(New Infinitary Mathematics)

Can semisets save vagueness?

Sometimes AST is promoted as a theory that is able to handle vagueness. I think this is not correct. We can use these examples as explanations, but the theory cannot be applied to practical situations where we need to handle vagueness. [...] the relation to vague concepts cannot be taken literally.

(P. Pudlák, Logical Foundations of Mathematics and Computational Complexity, 2013)

Can semisets save vagueness?

Most classes we encounter in the real world allow us to interpret them as semisets.

Examples of proper semisets are encountered everywhere.

Proper semisets are not rare occurrences, but rather among the most common phenomena.

(Bratislava, 1989)

We are going to show the existence of proper semisets.

The class of codes of all apes is a proper semiset. Our example is by no means isolated.

(Teubner, 1979)

Instrumentalist reading

Direct reflection of limitations connected to set theory.

- Greenness as a *primary* bearer of the phenomenon of non-sharpness
- The class of green things is merely the secondary bearer of the phenomenon of greenness.
- Secondary bearers provide a convenient tool for exploration of greenness

  > "After all, direct exploration of this phenomenon on its primary bearers has, so far, not been very successful."

- Whenever secondary bearers are missing, we invent convenient artificial secondary bearers (following Bolzano advice modified for our purposes).

(New Infinitary Mathematics)

New school of phenomenology?

- Merlau-Ponty: What is phenomenology? It may seem strange that we still have to ask this question half century after first works of Husserl. However, the question is far from being solved. (Phenomenology of Perception, 1945)
- J. Patočka: The reason why absolute subjectivity lost approach to itself is not answered by Husserl at all. [...] Phenomenology helps precisely because it is not a closed system. (Introduction to Husserl's Phenomenology, 1965)
- B. Hopkins: Husserl's phenomenological account of the formal structure of a mathematical manifold does not adequately distinguish the formality of meanings that are ideal from that of those that are formalized. (Horizons of Phenomenology, 2023)

New school of phenomenology?

It is not easy to add anything substantial to what is written in Husserl's work *The Crisis of the European Sciences and Transcendental Phenomenology*.

(P. Vopěnka, Geometrization of the real world, 1995)

Here we parted ways with Heidegger; we are not creating philosophy, but mathematics.

(P. Vopěnka, New Infinitary Mathematics, 2014)

The Theory
of Semisets

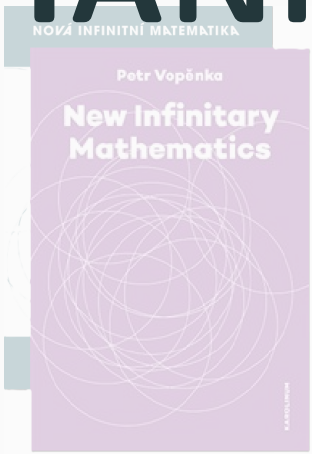
TEUBNER-TEXTE
zur Mathematik
Band 133
Schmeisser/Triebel
Function Spaces,
Differential Operators
and Nonlinear Analysis
B. G. Teubner Verlagsgesellschaft
Stuttgart · Leipzig


Úhelný kámen
evropské
vzdělanosti
a moci
Petr
Vopěnka
ROZPRAVY
S GEOMETRIÍ

Meditace
o základech
vědy

THANK YOU

Petr Vopěnka
Úvod do matematiky
v alternatívnej
teórii množín


NOVÁ INFINITNÍ MATEMATIKA
Petr Vopěnka
New Infinitary
Mathematics

Vyprávění
o kvůře
novobarokní
matematiky